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# Attitude Head Pursuit Transition Guidance Law

Yang Chunlei, Tang Shengjing\*, Shi Jiao, Guo Jie

*School of Aerospace Engineering, Beijing Institute of Technology, Beijing 100081, China*

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## Abstract

As an improved guidance method, the attitude head pursuit guidance (AHPG) law enables the attitude pursuit guidance (APG) law to be more suited to transition guidance of air-to-ground missiles. By adding a head angle into the attitude angle of APG, AHPG directs the missile axis onto the line of sight (LOS). The maximum range trajectory simulation shows that the elevator deflection angle reaches the saturated value of  $10^\circ$  at the outset and the impact angle is less than  $60^\circ$  when APG is used as transition guidance law. However, the elevator deflection angle on the whole trajectory is reduced to under  $5^\circ$  and the impact angle increased to over  $60^\circ$  when AHPG is used. The formulae to calculate head angles are derived for different target distributions. The simulation of multiple trajectories shows that with the help of the formulae based on AHPG law, the same performance could be achieved.

**Keywords:** flight vehicle design; attitude head pursuit method; trajectory calculations; transition guidance law; head angle; missiles

## 1. Introduction

Long range combined with high accuracy dominates the development of missile technology and draws strong attraction from missile engineers<sup>[1]</sup>. However, as they are always in contradiction with each other, rather than a single guidance method, a serial compound guidance<sup>[2]</sup> using different guidance methods on different flight phases is proposed to find out the balance between them. The terminal guidance law of a missile is typically proportional navigation<sup>[3–6]</sup> and the transition guidance law varies with the detected parameters. The transition guidance law is supposed to guide the missile to the target acquisition area of terminal guidance and transfer to the terminal flight phase smoothly<sup>[7]</sup>, so the work to be done by the transition guidance law is easy if the target acquisition area of terminal guidance is large.

As a guidance method, the attitude pursuit guidance (APG) law<sup>[8–10]</sup> is aimed to keep the longitudinal axis of missile body pointing at the target at all times by changing the flight path angle through altering the pitch angle. Usually, the longitudinal axis of the missile body lags behind the line of sight (LOS) and has precedence over the velocity vector because of the

inertial lag inherent in the missile system. This makes the response of the guidance system slow and the missile unable to make an effective hit against the maneuvering target<sup>[11]</sup>. But, since APG is cheap and easy to implement and, moreover, the transition guidance does not require high accuracy, APG can still be used as the transition guidance law to work together with other high-precision guidance methods for the terminal guidance.

In this article, while using APG as the transition guidance law for an air-to-ground missile, attitude head pursuit guidance (AHPG) law is proposed based on the flight parameter analysis, and the formulae for calculation of head angles are derived for different target distributions and finally a flight trajectory simulation is conducted.

## 2. Description of Problem

Fig.1 shows an air-to-ground missile ( $M$ ) that is to attack a target ( $T$ ) at a velocity  $v_T$  on the ground from the air at the initial height of approximately 4.5 km and a velocity  $v$  perpendicular to the ground. Its attack area is a circle with 4 km in radius on the ground. Making use of the serial compound guidance method, the seeker of the missile can achieve an accurate terminal proportional navigation, but has a short effective range as its defect. As a result, other detectors should be used to measure the LOS angle  $q$  when the missile is far away from the target.

\*Corresponding author. Tel.: +86-10-68911036  
E-mail address: [tangsj@bit.edu.cn](mailto:tangsj@bit.edu.cn)

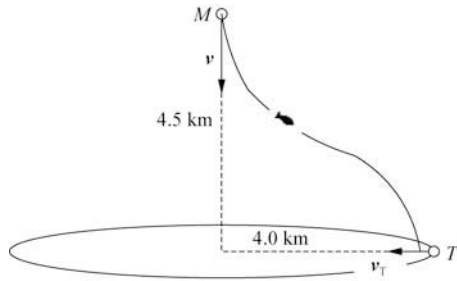


Fig.1 Attacking situation.

The missile is assumed to fly in a longitudinal plane and attack a faraway target. Given the fixed terminal guidance law, the transition guidance law should meet the requirements that the elevator deflection angle on whole trajectory be less than  $5^\circ$ , and the impact angle more than  $60^\circ$  and the elevator deflection angle do not suffer dramatic changes after the guidance laws switch.

### 3. APG Law

The equation of the APG law is  $\vartheta_c = q$ , namely the attitude angle command is equal to the LOS angle. The APG law needs to be achieved via attitude autopilot<sup>[12]</sup>, which is shown in Fig.2, where,  $\vartheta_c$  is the attitude angle command,  $K$  the angle error gain,  $k_s$  the servo gain,  $T_s$  the servo time constant,  $\delta_z$  the elevator deflection angle,  $k_\delta$  the missile steady state gain,  $T_i$  the missile angle of attack time constant,  $T_m$  the missile rotational mode time constant,  $\mu_m$  the damping ratio,  $K_g$  the rate gyro gain and  $\vartheta$  the pitch angle.

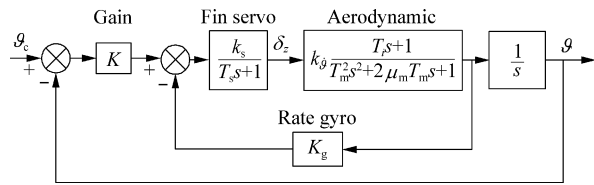
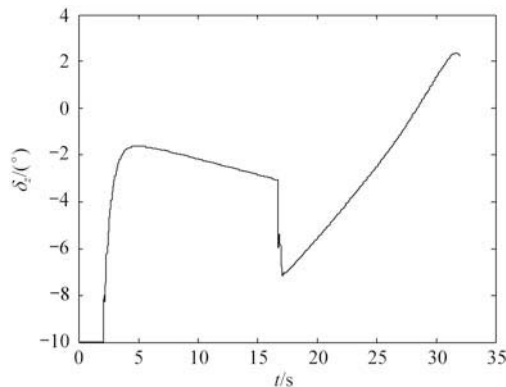
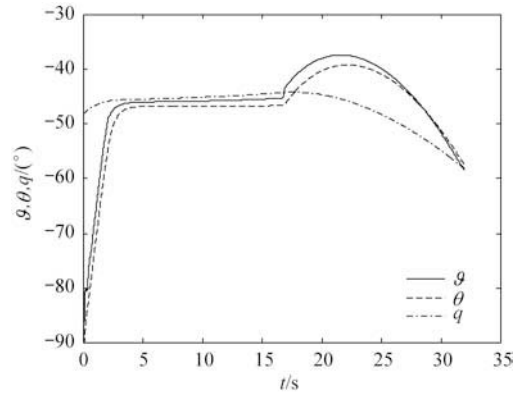


Fig.2 Principle diagram of attitude autopilot.

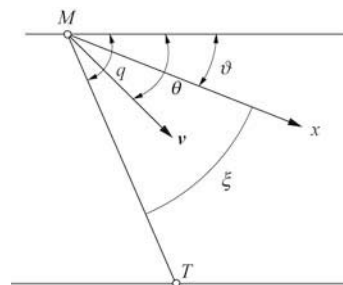
Fig.3 and Fig.4 show the curves of  $\delta_z$ ,  $\vartheta$ ,  $\theta$  and  $q$  in the flight trajectory simulation that uses APG as the transition guidance law where  $\theta$  is the flight path angle.

Fig.3 Curve of  $\delta_z$  in APG.Fig.4 Curves of  $\vartheta$ ,  $\theta$  and  $q$  in APG.

From Fig.3 and Fig.4, it could be noted that when APG is used as the transition guidance law,  $\delta_z$  reaches the  $-10^\circ$  saturated value at the initial time, which might cause the guidance loop to become unstable. Although  $\vartheta$  and  $\theta$  are able to follow up  $q$  quickly, the missile pulls up again at about  $t = 17$  s with big variance of  $\delta_z$  thus reducing the stability of the missile at the switching moment of the guidance law and the impact angle of about  $50^\circ$  would not make most of the air-to-ground missile height. The APG law, therefore, fails to meet the performance requirements.

### 4. AHPG Law

Suppose that at the time  $\tau$  the guidance law switches. In order to increase the impact angle, the missile velocity  $v$  of direction at  $\tau$  is required to be higher than the LOS ( $MT$ ), so that the trajectory can pull up at the beginning and plunge down by the end of the terminal guidance phase<sup>[13]</sup>. When the attitude angle is used as the control parameter, the longitudinal axis of the missile body  $Mx$  must be much higher than the LOS because the former is higher than the velocity direction. Let the angle between  $Mx$  and  $MT$  be denoted by  $\xi$ , and  $\vartheta = q + \xi$  should be tenable to ensure the smooth switch of the guidance law. Fig.5 illustrates the ideal relationship between the angles at  $\tau$ .

Fig.5 Ideal relationship between angles at  $\tau$ .

From the equation of APG  $\vartheta_c = q$ , it can be seen that  $\vartheta$  always lags behind  $q$  and cannot meet the equation  $\vartheta = q + \xi$  at  $\tau$ . So the head angle  $\eta$  is added to  $\vartheta_c$  to turn it into  $\vartheta_c = q + \eta$ . The increased  $\vartheta_c$  pulls the longitudi-

nal axis of missile body to the direction higher than the LOS<sup>[14-15]</sup>. This guidance law, which makes the longitudinal axis of missile body point over the LOS, is called AHPG law, as shown in Fig.6.

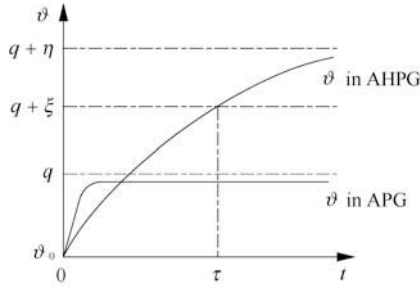


Fig.6 Curves of  $\theta$  in APG and AHPG.

By setting  $\eta=100^\circ$  and  $K=5$ , a flight trajectory simulation is conducted to use AHPG as the missile transition guidance law and obtain data sets of  $\delta_z$ ,  $\vartheta$ ,  $\theta$  and  $q$ , with which the curves are plotted (see Fig.7 and Fig.8).

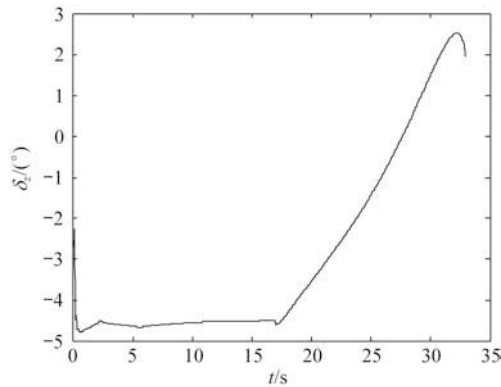


Fig.7 Curve of  $\delta_z$  in AHPG.

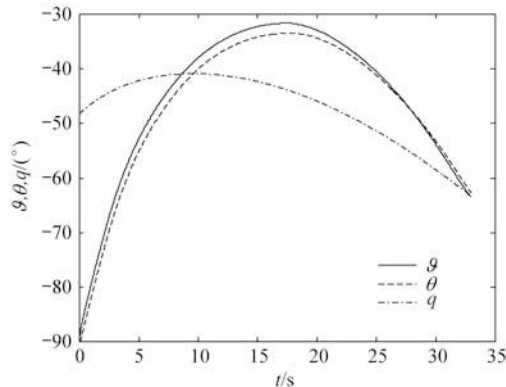


Fig.8 Curves of  $\vartheta$ ,  $\theta$ ,  $q$  in AHPG.

As shown in Fig.7 and Fig.8,  $\delta_z$  in AHPG is less than  $5^\circ$ , and  $\delta_z$ ,  $\vartheta$ ,  $\theta$  vary smoothly at the switching moment with the impact angle increasing to more than  $60^\circ$ .

$\eta$  qua an attitude command is not attained by means of measurement but rather calculation, so it does not pose requirements for detection equipment. However,

it affects the frame angle  $\psi$ , the angle between the missile body axis and the LOS. The transition guidance limits the frame angle to  $\pm 90^\circ$  and the terminal guidance  $\pm 30^\circ$ . Fig.9 evinces the curve of frame angle.

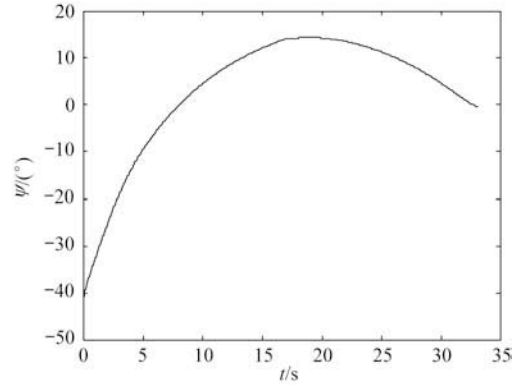


Fig.9 Curve of  $\psi$ .

It can be observed that the frame angles ranging from  $-42^\circ$  to  $14^\circ$  in transition guidance and  $15^\circ$  to  $-1^\circ$  in terminal guidance are both within the bounds. Moreover, the frame angle will become smaller if the target is nearer.

These results show that the AHPG law is more suitable for the transition guidance law of the air-to-ground missiles than the APG law. It decreases the required overload, provides better initial conditions for terminal guidance, achieves the stable transition between guidance laws, increases the impact angle and utilizes the air-to-ground missile's advantages. At the same time, the missile exhibits slow responses when the AHPG law is employed (see the curve of unit step response of attitude autopilot at  $t=0$  in Fig.10, where  $A$  is amplitude of unit step response). This means the AHPG law as the transition guidance law can only be applied to attacking targets with low maneuverability.

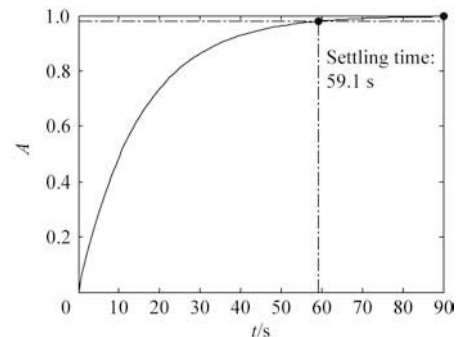


Fig.10 Unit step response of attitude autopilot at  $t=0$ .

## 5. Expression of Head Angle

When a missile attacks a maneuvering target, the latter may be located at any position in the seeker detection area. For the case under study, the target is assumed to appear in a circle with 4 km in radius, so the initial LOS angle  $q_0$  is about  $-48^\circ$  to  $-90^\circ$ . In order to

attack the target anywhere it is,  $\eta$  must be alterable. The method to determine  $\eta$  is introduced as follows.

To begin with, the attitude autopilot must be simplified. From Fig.2, as there is an integral element after the inner loop, the dynamic characteristics of the servo and the missile have little effect on the outer loop. Thus the servo and the missile can be simplified into an unchangeable element, and Fig.11 shows  $k_s$  and  $k_{\dot{\theta}}$ .

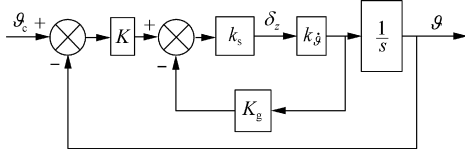


Fig.11 Principle diagram of simplified attitude autopilot.

Fig.12 shows the effects of simplification by comparing the unit step response curve of the original attitude autopilot at  $t = 0$  with the simplified one.

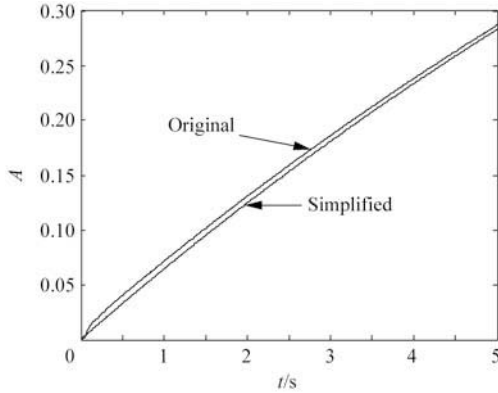


Fig.12 Comparison of unit step response curve of original attitude autopilot at  $t = 0$  with simplified one.

The comparison evidences the close similarity between the two responses, which proves the reliability of the simplification for computing the head angles.

Then  $K$  can be calculated. From Fig.11, can be derived

$$\delta_z = \frac{k_s(\theta_c - \theta)}{1 + k_s k_{\dot{\theta}} K_g} K \quad (1)$$

Eq.(1) indicates that the initial  $\delta_z$  is directly proportional to  $K$ , so the value of  $K$  must be limited to prevent  $\delta_z$  from reaching its saturation. Suppose there is  $\delta_z(^{\circ}) = pK$ ,  $p \in [0,1]$  in the air-to-ground missile; then  $K = 5$  is set to make sure the maximum  $\delta_z$  is less than  $5^{\circ}$ , keeping enough overload margin. The disadvantage of slow responding can be overcome by increasing  $\eta$ .

From Fig.11, the transfer function of attitude autopilot is as follows

$$\frac{\theta(s)}{\theta_c(s)} = \frac{1}{Ts + 1} \quad (2)$$

where

$$T = \frac{1 + K_g k_s k_{\dot{\theta}}}{K k_s k_{\dot{\theta}}} \quad (3)$$

in which  $T$  is the time constant of attitude autopilot, and  $K_g$  and  $k_s$  are determined by the terminal guidance law.

Eq.(2) shows that the attitude autopilot is almost a simple phase-lag element. From Fig.4 and Fig.8, it follows that as  $q$  changes little in the transition guidance phase and  $\eta$  keeps constant in each trajectory,  $\theta_c = q + \eta$  is also subject to minor changes. That is to say, the input signal of attitude autopilot is almost step input. The responding expression of the simple phase-lag element to step input is as follows:

$$\begin{aligned} \theta_t &= \theta_0 + (\theta_c - \theta_0)(1 - e^{-t/T}) \approx \\ &\theta_0 + (q_0 + \eta - \theta_0)(1 - e^{-t/T}) \end{aligned} \quad (4)$$

where  $\theta_0$  is the initial pitch angle, and  $t = 1, 2, \dots$ .

The switching moment  $\tau$  varies with respect to the target distance:  $\tau$  is large if the target is remote and slims down when it comes nearer. Also from Eq.(3), this is true of  $T$  with respect to trajectory and flight time. As for the ratio of  $\tau/T$ , it almost remains the same in all trajectories. For the missile under study, since  $\tau/T \in [0.37, 0.47]$ , it could be chosen as 0.42, and Eq.(4) can be written into

$$\begin{aligned} \theta_{\tau} &= \theta_0 + (q_0 + \eta - \theta_0)(1 - e^{-\tau/T}) = \\ &0.343q_0 + 0.343\eta + 0.657\theta_0 \end{aligned} \quad (5)$$

As mentioned above, the following relationship holds at  $\tau$ :

$$\theta_{\tau} = q_{\tau} + \xi \approx q_0 + \xi \quad (6)$$

The value of  $\xi$  is reliant on the LOS angle at  $\tau$  and  $q_{\tau}$ , and  $\xi = 0$  when  $q_{\tau} = -\pi/2$  and  $\xi$  increases as  $q_{\tau}$  rises, so

$$\xi = (q_0 + \pi/2)/n \quad (7)$$

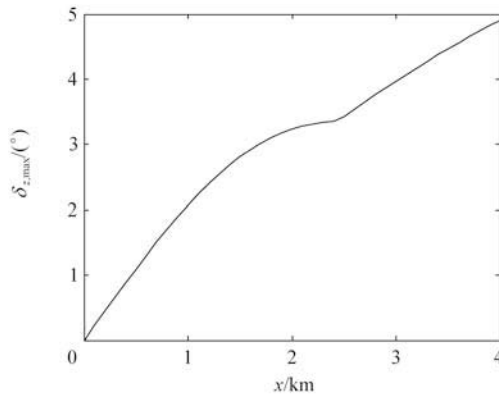
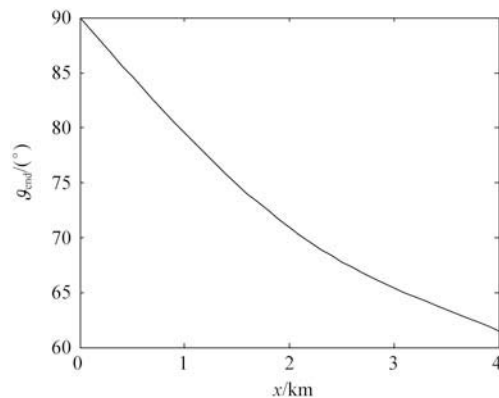
where  $n$  depends upon concrete missiles. For the missile under study,  $n = 5$ .

By combining Eq.(5), Eq.(6) and Eq.(7),  $\eta$  can be found to be

$$\eta = 2.4985q_0 - 1.9155\theta_0 + 0.2915\pi \quad (8)$$

It can be seen from Eq.(8) that  $\eta$  varies as a function of initial attitude of the missile and initial position of the target. Assuming that the position of the target varies from 0 to 4.0 km, the maximum elevator deflection angles  $\delta_{z,\max}$  and the impact angles  $\theta_{\text{end}}$  of each trajectory can be found out at each target position through simulations. The curves in Fig.13 and Fig.14 present the results respectively.

From them, it can be discovered that  $\delta_{z,\max}$  is always under  $5^{\circ}$  and  $\theta_{\text{end}}$  over  $60^{\circ}$ , which evidences the correctness of the expression of the head angle and its feasibility to calculate the head angles in the air-to-ground missile's attitude head pursuit transition guidance law.

Fig.13  $\delta_{z,max}$  vs target position.Fig.14  $\theta_{end}$  vs target position.

## 6. Conclusions

As the flight parameters of the APG law fails to satisfactorily meet the performance requirements when used as the transition guidance law, the AHPG law is developed by adding in a head angle into the attitude angle command. Simulation of a typical trajectory shows that the impact angle  $\theta_{end}$  keeps more than  $60^{\circ}$  and the elevator deflection angle less than  $5^{\circ}$  on the whole trajectory. Derived on the basis of different target distributions, the formulae to calculate head angles are proved to be suitable for the AHPG law by the results of the multiple trajectory simulations.

The AHPG law is superior in greater guidance stability with the attitude angles meeting the requirement for the terminal guidance, but deficient in quite slow variation of the missile attitude. The AHPG law cannot be used as the terminal guidance law or to attack the target with high maneuverability because of its low guidance accuracy; consequently, it can only be applied to the transition guidance phase, in which the LOS angle changes little and the transition guidance time is large.

Nevertheless, the AHPG law and the method of determining head angles are of practical use in designing guidance law for the air-to-ground missiles.

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## Biography:

**Yang Chunlei** Born in 1983, he received B.S. degree in 2005, and now he is working toward the Ph.D. degree in Beijing Institute of Technology. His main research interests include flight vehicle design and flight dynamics.  
E-mail: henryja@bit.edu.cn